ASCHAM SCHOOL
2008

# MATHEMATICS EXTENSION 1: YEAR 12 TRIAL EXAMINATION

TIME ALLOWED: 2 HOURS PLUS 5 MINUTES' READING TIME

EXAMINATION DATE: MONDAY 28 JULY

**INSTRUCTIONS**

ALL QUESTIONS MAY BE ATTEMPTED.
ALL QUESTIONS ARE OF EQUAL VALUE (12 MARKS).
ALL NECESSARY WORKING MUST BE SHOWN.
MARKS MAY NOT BE AWARDED FOR CARELESS WORK.
APPROVED CALCULATORS AND TEMPLATES MAY BE USED.

**COLLECTION**

START EACH QUESTION IN A NEW BOOKLET.
IF YOU USE A SECOND BOOKLET FOR A QUESTION, PLACE IT INSIDE THE FIRST.
WRITE YOUR NAME, TEACHER'S NAME AND QUESTION NUMBER ON EACH BOOKLET.

JMH

***Standard Integrals***

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

## Question 1

(a) Evaluate $\int_0^1 \frac{dx}{\sqrt{4-x^2}}$. **2**

(b) Solve the inequality $\frac{1}{x^2} \geq 4$. **2**

(c) Find the derivative of $e^{\sin 3x}$. **2**

(d) Evaluate $\int_{\frac{1}{2}}^{1} 4t(2t-1)^3\,dt$ by using the substitution $u = 2t - 1$. **4**

(e) Use the *t*-results to show that $\cot\theta + \tan\frac{1}{2}\theta = \operatorname{cosec}\theta$. **2**

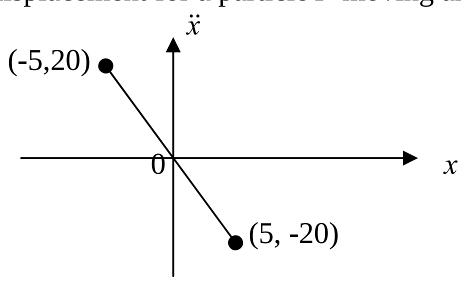
## Question 2 Begin a new booklet.

(a) Find all solutions to the equation $\cos\theta = -\frac{1}{2}$. **2**

(b) (i) On the same diagram sketch the graphs of $y = |x-2|$ and $y = \frac{1}{2}x - 1$. **2**

(ii) By using (i) or otherwise, determine the value of $c$ such that the equation $|x-2| = \frac{1}{2}x + c$ has exactly 2 solutions. **1**

(c) Consider the graph of $\ddot{x}$ versus $x$ shown below, where $\ddot{x}$ is acceleration and $x$ is displacement for a particle $P$ moving along the $x$-axis.




(i) Explain why the equation relating $\ddot{x}$ and $x$ is given by $\ddot{x} = -4x$. **1**

(ii) Explain why $P$ is moving in simple harmonic motion. **1**

(iii) From the graph, find the amplitude of the motion. **1**

(iv) Find the period of the motion. **1**

(v) Find the equation of the motion in the form $x = a\sin(nt+\varepsilon)$ if $x = 2.5$ when $t = 0$. Assume $-\frac{\pi}{2} \le \varepsilon \le \frac{\pi}{2}$. **2**

(vi) Find the maximum speed of $P$. **1**

## Question 3 Begin a new booklet.

(a) (i) State the domain of $y = \ln(1-x)$. Hence sketch the graph of $y = \ln(1-x)$. **2**

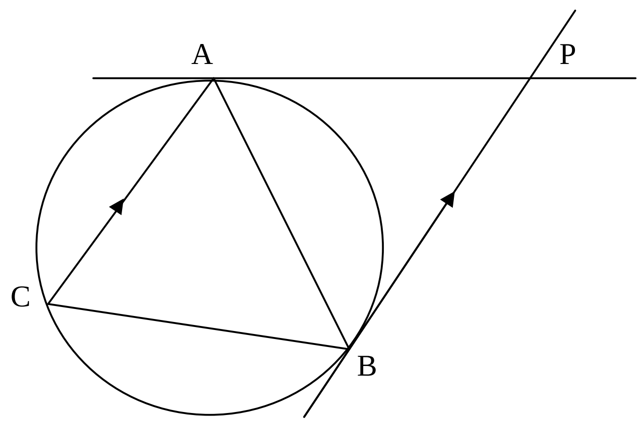
A particle $P$ moves along the $x$-axis with velocity $v$ cm/s and displacement $x$ cm at $t$ seconds, according to the equation $v = -e^{-x}$.

(ii) Derive an equation for $x$ in terms of $t$, given that when $t = 0$, $x = 0$. Hence show $x = \ln(1-t)$. **3**

(iii) Find the time interval over which $P$ moves. **1**

(iv) With reference to part (i) or otherwise, explain why the particle is speeding up for all $t$ in the domain. **1**

(b)




Consider the points *A, B* and *C,* lying on the circle, with tangents drawn from *A* and *B*, meeting at *P*. *AC//BP.*
Copy the diagram.

(i) Prove $AB = BC$. **2**

(ii) Prove $\angle ABC = \angle APB$. **3**

## Question 4 Begin a new booklet

(a)

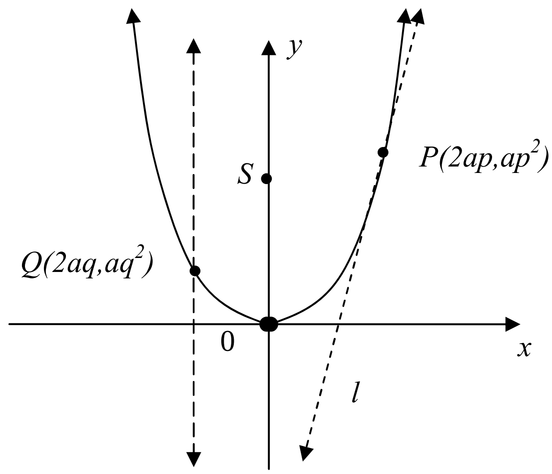



Diagram not to scale.

Two points $P\left(2ap, ap^2\right)$ and $Q\left(2aq, aq^2\right)$ lie on the parabola $x^2 = 4ay$.
Copy the diagram.

(i) Show that the equation of the tangent $l$ to the parabola at the point $P$ is $y = px - ap^2$. **2**

(ii) The tangent at $P$ and the line through $Q$ parallel to the $y$-axis intersect at $T$. Show that the coordinates of $T$ are $\left(2aq, 2apq - ap^2\right)$. **2**

(iii) Find the coordinates of $M$, the midpoint of $PT$. **1**

(iv) Hence or otherwise find the Cartesian equation of the locus of $M$ when $pq = -1$. **1**

(b) Salinity is a major problem confronting the environment. The quantity of salt $Q$ kg in a reservoir of water after $t$ days can be modelled by the equation $Q = 1000\left(1 - e^{-0.01t}\right)$.

(i) Show that $Q = 1000\left(1 - e^{-0.01t}\right)$ satisfies the differential equation $\frac{dQ}{dt} = 0.01(1000 - Q)$. **2**

(ii) What is the initial quantity of salt in the reservoir? **1**
(iii) Find the rate at which the salt is changing after 10 days. **1**
(iv) What will happen to the salt in the reservoir as time goes on? **1**
(v) Sketch the equation $Q = 1000\left(1 - e^{-0.01t}\right)$. **1**

**Question 5** **Begin a new booklet.**

(a) Use one application of Newton's method to find an approximation to the root of $\cos x + \sin x - x = 0$, nearest to $x = 1.2$ . **3**

(b) (i) Express $\sin x - \cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 \le \alpha \le \pi$ . **2**

(ii) Hence solve the equation $\sin x - \cos x = 1$, for $0 \le x \le 2\pi$ . **2**

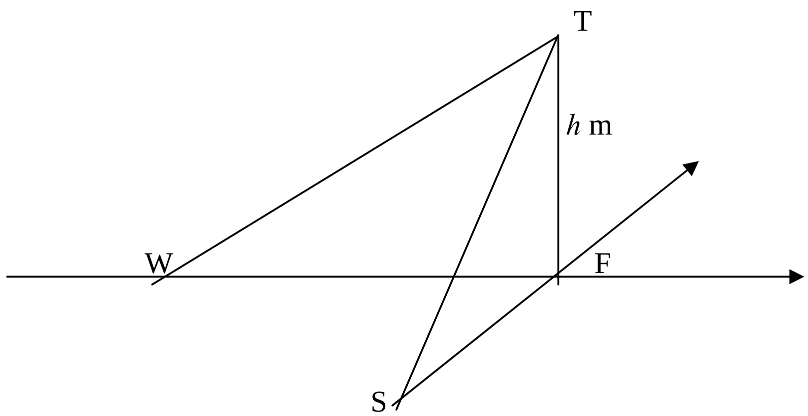
(c) (i) For what values of $x$ is $\sin^{-1} x$ defined? **1**

(ii) Find the maximum value of $2x(1 - x)$. **2**

(iii) Find the range of the function $f$ given by $f(x) = \sin^{-1}\{2x(1 - x)\}$ with domain $0 \le x \le 1$. **2**

**Question 6** **Begin a new booklet.**

(a)




A surveyor estimates the height *h* m of a mesa in the desert by taking two readings. Standing due South of the foot *F* of the mesa at point *S*, the angle of elevation to the top of the mesa is 9º, whereas from point *W,* due west of the foot of the mesa, the angle of elevation is 15º to the top of the mesa. The surveyor knows that the distance from *S* to *W* is 1200 m.
Copy the diagram.

(i) Show that $h^2\left(\cot^2 9° + \cot^2 15°\right) = 1200^2$. **2**

(ii) Hence find the height of the mesa correct to two significant figures. **1**

(b) Prove by induction that $\dfrac{1}{x-1} - \dfrac{1}{x} - \dfrac{1}{x^2} - ... - \dfrac{1}{x^n} = \dfrac{1}{x^n\left(x-1\right)}$, for all positive integers *n* and $x \neq 0$ , 1. **4**

(c) A goose is flying horizontally along at a speed of *S* m/s and at an altitude of *H* m when it passes a goose hunter below lying at (0,0) with a goose rifle. Simultaneously, the hunter shoots a bullet up at an angle $\theta$ from the horizontal with a velocity *V* m/s hoping to shoot the goose. Assume the acceleration due to gravity is *g* m/s$^2$.

(i) Explain why $x = St$ describes the distance the goose has flown after *t* seconds. **1**

You may assume the equations of motion $x = Vt\cos\theta$ and $y = Vt\sin\theta - \dfrac{gt^2}{2}$ describe the horizontal and vertical distances of the bullet after *t* seconds.

Assume that the bullet hits the goose.

(ii) Explain why $S = V\cos\theta$. **1**

(iii) Hence show that $H = -\dfrac{gt^2}{2} + St\tan\theta$. **1**

(iv) Hence show that there are two possible occasions when the bullet can shoot the goose if $S^2\tan^2\theta > 2gH$. **2**

## Question 7 Begin a new booklet

(a) A sector maintains a constant area of $50\pi \text{ cm}^2$, while the radius $r$ cm and angle at the centre $\theta$ change. Find the rate at which the radius is changing given that $\frac{d\theta}{dt} = \frac{\pi}{5}$ radian/second, when the radius is 10cm. **4**

(b) (i) Prove that $y = \frac{1}{x}$ is concave up for all $x > 0$. **3**

(ii) Sketch $y = \frac{1}{x}$ for $x > 0$. **1**

Suppose $0 < a < b$ and consider the points $A\left(a, \frac{1}{a}\right)$ and $B\left(b, \frac{1}{b}\right)$ on the graph of $y = \frac{1}{x}$.

(iii) Find the coordinates of the point $P$ that divides the line segment $AB$ in the ratio 2:1. **2**

(iv) With reference to areas and by careful argument, **2**

deduce that $\ln\frac{b}{a} < \frac{b^2 - a^2}{2ab}$.

**SOLUTIONS**

ASCHAM TRIAL EXT 1 MATHS Yr12 2008 SOLUTIONS

1. a) $\int_0^1 \frac{dx}{\sqrt{4-x^2}} = \left[\sin^{-1}\frac{x}{2}\right]_0^1$

(2) $= \sin^{-1}\frac{1}{2} - \sin^{-1}\frac{0}{2}$

$= \frac{\pi}{6}$

b) $\frac{1}{x^2} \geq 4$ , $x \neq 0$

$\frac{1}{4} \geq x^2$ (2)

$-\frac{1}{2} \leq x \leq \frac{1}{2}$ , $x \neq 0$

c) $\frac{d}{dx}\left(e^{\sin 3x}\right) = 3\cos 3x\, e^{\sin 3x}$ (2)

d) $\int_{\frac{1}{2}}^{1} 4t(2t-1)^3\,dt$ | $u = 2t-1$, $du = 2dt$, $t=1$ $u=1$, $t=\frac{1}{2}$ $u=0$

$= \int_0^1 2(u+1)(u)^3 \cdot \frac{du}{2}$

$= \int_0^1 u^4 + u^3\,du$ (4)

$= \left[\frac{u^5}{5} + \frac{u^4}{4}\right]_0^1$

$= \frac{1}{5} + \frac{1}{4} - 0$

$= \frac{9}{20}$

e) $\cot\theta + \tan\frac{1}{2}\theta = \frac{1}{\tan\theta} + \tan\frac{1}{2}\theta$

$= \frac{1-t^2}{2t} + t$ where $t = \tan\frac{\theta}{2}$

$= \frac{1-t^2+2t^2}{2t}$

$= \frac{1+t^2}{2t}$ (2)

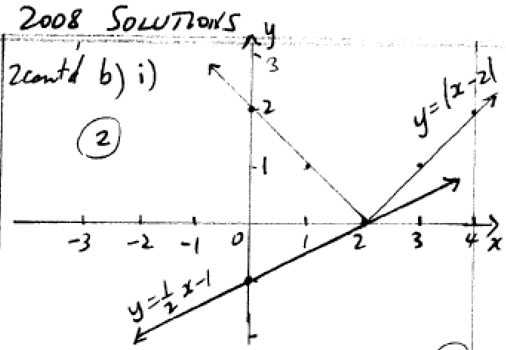
$= \frac{1}{\sin\theta}$

$= \operatorname{cosec}\theta$ . QED.

2 a) $\cos\theta = \frac{-1}{2}$ (2)

$\theta = 2\pi n \pm \frac{2\pi}{3}$ , $n \in \mathbb{Z}$.

2 cont'd b) i) (2)




ii) $c > -1$ as intersects twice. (1)

c) i) $m = \frac{-20}{5}$ (1)

$= -4$ $\therefore$ Equation $\ddot{x} = -4x$.

ii) SHM since in form $\ddot{x} = -n^2x$ (1) where $n = 2$. (1)

iii) $-5 \leq x \leq 5$ $\therefore$ Amplitude = 5 unit.

iv) Period $= \frac{2\pi}{n}$

$= \frac{2\pi}{2}$ (1)

$= \pi$

v) $x = a\sin(2t+\varepsilon)$ $x = 2.5$, $t = 0$

$2.5 = 5\sin(2t+\varepsilon)$

$\frac{2.5}{5} = \sin(2\times 0+\varepsilon)$

$\frac{1}{2} = \sin\varepsilon$ (2)

$\varepsilon = \frac{\pi}{6}$

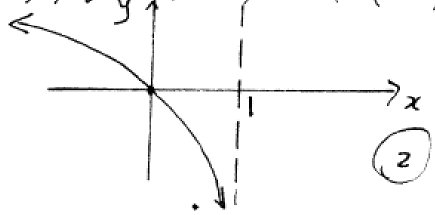
$\therefore$ $x = 5\sin\left(2t+\frac{\pi}{6}\right)$

vi) Max speed : $\dot{x} = 10\cos\left(2t+\frac{\pi}{6}\right)$

occurs when $\cos\left(2t+\frac{\pi}{6}\right) = 1$

$\therefore$ Max Speed = 10 . (1)

3. a) i) $y = \ln(1-x) = \ln(-(x-1))$




(2)

ASCHAM TRIAL EXTENSION 1 MATHS SOLUTIONS YR 12 2008 cont'd

3 cont'd a) ii) $v = -e^{-x}$ $t = 0$ $x = 0$

$\frac{dx}{dt} = -e^{-x}$

$\frac{dt}{dx} = -e^{x}$

$t = \int -e^{x}\,dx$ (3)

$t = -e^{x} + C$

$0 = -e^{0} + C$

$C = 1$

$\therefore t = -e^{x} + 1$

$\therefore e^{x} = 1 - t$

$\therefore x = \ln(1 - t)$.

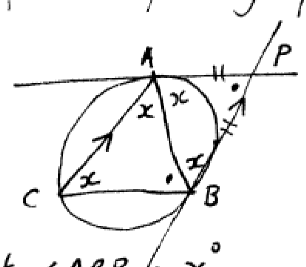
iii) Domain $1 - t > 0$

$\therefore 0 < t < 1$. (1)

iv) By (i) graph, decreasing and concave down $\therefore v < 0$ and $a < 0$

$\therefore$ particle speeding up. (1)

b)




Let $\angle ABP = x^{\circ}$.

i) RTP: $AB = BC$ ie isosceles

Proof: $\angle CAB = x^{\circ}$ (alternate $\angle$s, $AC \parallel PB$)

$\angle BCA = x^{\circ}$ ($\angle$ between tangent & chord = $\angle$ in alternate segment) (2)

$\therefore AB = BC$ (sides opposite equal $\angle$s equal)

ii) RTP: $\angle ABC = \angle APB$

Proof: $PA = PB$ (tangents from external pt equal)

$\therefore \angle PAB = x^{\circ}$ (base $\angle$s of isosceles $\Delta APB$) (3)

$\therefore \angle ABC = \angle APB$ (both equal to $180^{\circ} - 2x$) ($\angle$ sum of $\Delta$).

4. a) i) $y = \frac{x^2}{4a}$ $y' = \frac{2x}{4a} = \frac{x}{2a}$

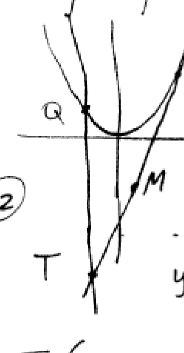
At $P$, $y' = \frac{2ap}{2a} = p$.

$\therefore y - ap^2 = p(x - 2ap)$ (2)

$\therefore y - ap^2 = px - 2ap^2$

$\therefore y = px - ap^2$.

ii)




(2)

$x$-coord of $Q$ is $2aq$.

$\therefore$ equation of $QT$ is $x = 2aq$.

$\therefore$ Sub into tangent:

$y = p(2aq) - ap^2$

$= 2apq - ap^2$

$\therefore T(2aq, 2apq - ap^2)$.

iii) $M\left(\frac{2aq + 2ap}{2}, \frac{2apq - ap^2 + ap^2}{2}\right)$

$M = (a(p+q), apq)$. (1)

iv) $pq = -1$ $\therefore M(a(p+q), -a)$

$\therefore x = a(p+q)$, $y = -a$ (1)

$\therefore$ Cartesian eqn is $y = -a$ (the directrix)

b) $Q = 1000(1 - e^{-0.01t})$

i) $\frac{dQ}{dt} = 1000(-0.01 \times -e^{-0.01t})$

Since $Q = 1000 - 1000e^{-0.01t}$

$1000e^{-0.01t} = 1000 - Q$

$\therefore \frac{dQ}{dt} = 0.01 \times 1000e^{-0.01t}$

$= 0.01(1000 - Q)$ (2)

ii) When $t = 0$ $Q = 1000(1 - e^{0}) = 0$ kg.

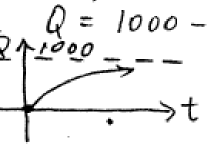
iii) $t = 10$, $\frac{dQ}{dt} = 0.01 \times 1000e^{-0.01 \times 10}$

$\doteqdot 9.05$ kg/day. (1)

iv) As $t \to \infty$, $e^{-0.01t} \to 0$ (1)

$\therefore Q = 1000 - 0$, approaches 1000kg.

v)




(1)

ASCHAM TRIAL MATHS EXT 1 SOLUTIONS YR 12 2008 cont'd

**5. a)** Let $f(x) = \cos x + \sin x - x$

$f'(x) = -\sin x + \cos x - 1$

$f(1.2) = \cos 1.2 + \sin 1.2 - 1.2 \doteqdot 0.09439684$

$f'(1.2) = -\sin 1.2 + \cos 1.2 - 1 = -1.569681331$

$\therefore x_2 = 1.2 - \dfrac{\quad}{\quad}$ (3)

$\doteqdot 1.260$

**b) i)** $\sin x - \cos x \equiv R\sin(x-\alpha)$

$\therefore \sin(x-\alpha) \equiv \frac{1}{R}\sin x - \frac{1}{R}\cos x$

$\therefore \sin x\cos\alpha - \cos x \sin\alpha \equiv \frac{1}{R}\sin x - \frac{1}{R}\cos x$

$\therefore \cos\alpha = \frac{1}{R}, \ \sin\alpha = \frac{1}{R}$

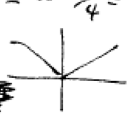

$\therefore \alpha = \frac{\pi}{4}, \ R = \sqrt{2}$ (2)

$\therefore \sin x - \cos x \equiv \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$.

**ii)** $\sin x - \cos x = 1 \quad 0 \le x \le 2\pi$

$\sqrt{2}\sin\left(x - \frac{\pi}{4}\right) = 1 \quad -\frac{\pi}{4} \le x - \frac{\pi}{4} \le \frac{7\pi}{4}$

$\sin\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

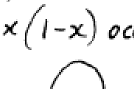


$\therefore x - \frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4}$

$\therefore x = \frac{\pi}{2}, \pi$. (2)

**c) i)** $\sin^{-1}x$ defined for $-1 \le x \le 1$. (1)

**ii)** Max. value of $2x(1-x)$ occurs when $x = \frac{1}{2}$:




$2\left(\frac{1}{2}\right)\left(1 - \frac{1}{2}\right) = \frac{1}{2}$.

$\therefore$ Max value $= \frac{1}{2}$. (2)

**iii)** $f(x) = \sin^{-1}\{2x(1-x)\}$ in $0 \le x \le 1$. Normal Range of $\sin^{-1}\square$: $-\frac{\pi}{2} \le \sin^{-1}\square \le \frac{\pi}{2}$

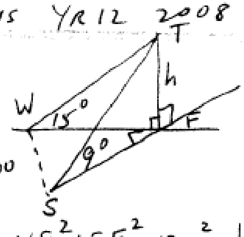
but for $0 \le x \le 1$: $0 \le \sin^{-1}\square \le \frac{\pi}{2}$

But max value of $2x(1-x) = \frac{1}{2}$

$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$; $\sin^{-1}0 = 0$

$\therefore 0 \le \sin^{-1}(2x(1-x)) \le \frac{\pi}{6}$ (2)

**6.**




**i)** $WF^2 + SF^2 = 1200^2$ by Pythagoras

$\tan 15° = \frac{h}{WF} \ \therefore WF = \frac{h}{\tan 15}$

$\therefore WF = h\cot 15$ in $\Delta WFT$

Similarly, $SF = h\cot 9°$ from $\Delta FST$

$\therefore$ Substituting: $(h\cot 15°)^2 + (h\cot 9°)^2 = 1200^2$

$\therefore h^2(\cot^2 9° + \cot^2 15°) = 1200^2$ (2)

**ii)** $\therefore h = \dfrac{1200}{\sqrt{\cot^2 9° + \cot^2 15°}}$ $(h > 0)$

$\doteqdot 160$ (2 s.f.) (1)

**b)** RTP: $\dfrac{1}{x-1} - \dfrac{1}{x} - \dfrac{1}{x^2} - \ldots - \dfrac{1}{x^n} = \dfrac{1}{x^n(x-1)}$.

Let $P(n)$ be the proposition:

Proof: $P(1)$: $\dfrac{1}{x-1} - \dfrac{1}{x} = \dfrac{x-(x-1)}{x^1(x-1)}$

$= \dfrac{x - x + 1}{x(x-1)}$

$= \dfrac{1}{x(x-1)}$

$\therefore P(1)$ true.

Assume $P(k)$ true ie:

$\dfrac{1}{x-1} - \dfrac{1}{x} - \dfrac{1}{x^2} - \ldots - \dfrac{1}{x^k} = \dfrac{1}{x^k(x-1)}$.

RTP: $P(k+1)$ true ie.

$\dfrac{1}{x-1} - \dfrac{1}{x} - \dfrac{1}{x^2} - \ldots - \dfrac{1}{x^k} - \dfrac{1}{x^{k+1}} = \dfrac{1}{(x^{k+1})(x-1)}$

Proof: Consider the LHS of $P(k+1)$:

$\dfrac{1}{x-1} - \dfrac{1}{x} - \dfrac{1}{x^2} - \ldots - \dfrac{1}{x^k} - \dfrac{1}{x^{k+1}} = \dfrac{1}{x^k(x-1)} - \dfrac{1}{x^{k+1}}$ (by assumption P(k))

$= \dfrac{x - (x-1)}{x^k(x-1) \cdot x^1}$

$= \dfrac{x - x + 1}{x^{k+1}(x-1)}$ (4)

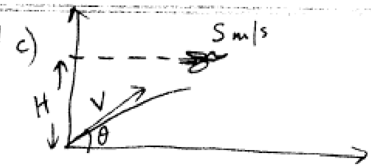
$= \dfrac{1}{x^{k+1}(x-1)}$

$=$ RHS of $P(k+1)$.

$\therefore$ Truth of $P(k) \Rightarrow$ Truth of $P(k+1)$ but $P(1)$ also true $\therefore P(n)$ true $\forall n \ge 1$ by Math. Ind.

ASCHAM TRIAL EXT 1 MATHS YR 12 2008 Solutions cont'd.

**6** cont'd c)




i) $x = \text{dist} \times \text{time}$ (1)

$\therefore x = St$ for the goose.

ii) If bullet hits goose then horizontal distances must be equal

$\therefore \quad Vt\cos\theta = St$ (1)

$\therefore \quad S = V\cos\theta$.

iii) $y = Vt\sin\theta - \frac{gt^2}{2}$ Vertical distances must be equal:

$H = Vt\sin\theta - \frac{gt^2}{2} \Rightarrow V = \frac{S}{\cos\theta}$ Sub

$\therefore H = \frac{S}{\cos\theta} t\sin\theta - \frac{gt^2}{2}$

$\therefore H = St\tan\theta - \frac{gt^2}{2}$ (1)

iv) Solve for $t$:

$gt^2 + 2H - 2St\tan\theta = 0$

$t = \frac{2S\tan\theta \pm \sqrt{4S^2\tan^2\theta - 4g.2H}}{2g}$

$\therefore \Delta > 0 \therefore 4S^2\tan^2\theta - 8gH > 0$

(2) $\therefore \quad S^2\tan^2\theta > 2gH$.

b) i) $y = x^{-1}$

$y' = -x^{-2}$

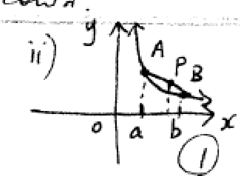
$y'' = 2x^{-3}$

$= \frac{2}{x^3} > 0$ (3)

for all $x > 0$

$\therefore$ Concave up.

ii)




(1)

$A\left(a, \frac{1}{a}\right)$

$B\left(b, \frac{1}{b}\right)$.

iii) $P\left(\frac{a \times 1 + b \times 2}{2+1}, \frac{\frac{1}{a} \times 1 + \frac{1}{b} \times 2}{2+1}\right)$

$= \left(\frac{a+2b}{3}, \frac{\frac{1}{a} + \frac{2}{b}}{3}\right)$ (2)

$= \left(\frac{a+2b}{3}, \frac{b+2a}{3ab}\right)$

iv) Area under curve $= \int_a^b \frac{1}{x}\,dx$

$= \left[\ln x\right]_a^b$

$= \ln b - \ln a$

$= \ln\frac{b}{a}$ unit².

Area of trapezium $= \frac{h}{2}(x+y)$

$= \frac{b-a}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$

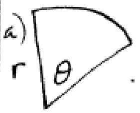
$= \left(\frac{b-a}{2}\right)\left(\frac{b+a}{ab}\right)$

$= \frac{b^2-a^2}{2ab}$ unit²

Since Area$_{\text{trapezium}}$ > Area under curve (Concave up)

then $\ln\frac{b}{a} < \frac{b^2-a^2}{2ab}$. (2)

**7.** a)




$A = \frac{1}{2}r^2\theta = 50\pi$.

$\therefore \frac{1}{2}r^2\theta = 50\pi$

$\theta = 100\pi r^{-2}$

$\therefore \frac{d\theta}{dr} = -200\pi r^{-3}$

$= \frac{-200\pi}{r^3}$

$\therefore \frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt}$

$= \frac{r^3}{-200\pi} \times \frac{\pi}{5}$ $\quad r = 10$

$= \frac{1000}{-200\pi} \times \frac{\pi}{5}$ (4)

$= -1$ cm/s.

$\therefore$ decreasing at 1cm/s.